An extention of the Holley-Liggett method for the one-dimensional contact process

1次元コンタクトプロセスに対する Holley-Liggett 法の拡張

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The contact process (CP) is a $\{0,1\}^{Z^d}$-valued continuous Markov process. In the mathematical field, this process is first introduced by T.E.Harris (1974). The CP can be interpreted as a simplest model for spread of an infection disease.

In this talk, we report on the result of the estimates of survival probability of the one-dimensional contact process. Holley-Liggett (1978) used renewal measure to obtain the upper bound for critical infection rate and the lower bound for the survival probability.

On the other hand, Katori-Konno (1990) introduced K-type correlation functions to estimate the survival probability of one-dimensional contact process and studied how to obtain such a new measure. They could analyze by an appropriate decoupling for some equations including the first equation of hierarchy of K-type correlation identities that holds between K-type correlation functions for invariant measure of one-dimensional contact process.

In the present talk, we extend renewal decoupling further and used K-type correlation functions that introduced one-point and two-point correlation functions with parameter $p \in [0,1]$.

When $p = 1$ (resp. $p = 1/2$), our result is the same as the result given by Holley-Liggett (1978)(resp. Katori-Konno (1990)).

As the result, it is shown that

(1)　$0 < p < 1/3$: the value of λ is not suitable as the upper bound.

(2)　$p = 0 : \lambda = 2$ coincides with the result of first upper bound on the critical value given by Holley-Liggett.

(3)　$1/3 \leq p \leq 1$: when $p = 1$, it becomes pair approximation between the distances of particles. when $p = 1$, the value of $\lambda = 1.84715$ is not better than the result of $\lambda = 1.78989$ (for $p = 1/2$), but is better than the value of $\lambda = 2$ given by Holley-Liggett.

Let $\rho_\lambda = E_{\nu_\lambda}(\eta(x))$ be the survival probability of the one-dimensional CP with $\rho_\lambda = 0$ for $0 \leq \lambda \leq \lambda_c$ and $\rho_\lambda > 0$ for $\lambda > \lambda_c$.

Holley-Liggett (1978) and Katori-Konno (1990) used the renewal measure as the approximation of the upper invariament measure to estimate the survival probability ρ_λ of the one-dimensional CP.

In the present talk, we introduce a new decoupling procedure of correlation functions and estimates of ρ_λ. Moreover we study how ρ_λ changes as parameter p changes.

References

[1]Holley,R.and Liggett,T.M.,The survival of contact process, Annals of Probability, Vol.6(1978), pp.198-206.

[2] Katori,M.,and Konno,N.,Phase transition in stationary states of contact process, Proceedings of the Institute of Statistical Mathematics Vol.38, No.2,(1990), pp.243-256 (in Japanese).