ABSTRACT

性転換を伴う生物個体群の存続性に関する数理モデル解析

Persistence of Population with Age-dependent Sex Reversal

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We consider the age-structured single-species population dynamics with age-dependent sex reversal from male to female. In our mathematical considerations, we focus the population persistence affected by the sex reversal, assuming that the sex reversal from male to female could be occuerd just after the critical age a_c with a constant sex reversal rate.

We denote the age distribution function of male age a and that of female age a' at time t by $v(a,t)$ and $w(a,t)$ respectively. In addition, $c(a,a',t)$ gives the density of reproductive couple which consists of male with age a and female with age a' at time t. We construct at first the dynamical system for the distribution of age-structured population with the sex reversal:

$$
\begin{aligned}
\frac{\partial v(a,t)}{\partial t}+\frac{\partial v(a,t)}{\partial a} &= -\delta_{11}v(a,t)-M(a)v(a,t)\\
\frac{\partial w(a',t)}{\partial t}+\frac{\partial w(a',t)}{\partial a'} &= -\delta_{22}w(a',t)+M(a')v(a',t)\\
\frac{\partial c(a,a',t)}{\partial t}+\frac{\partial c(a,a',t)}{\partial a}+\frac{\partial c(a,a',t)}{\partial a'} &= -(\delta_{11}+\delta_{22})c(a,a',t)\\
&\quad -D\cdot c(a,a',t)+e\cdot\left\{v(a,t)-\int_0^\infty c(a,a',t)da'\right\}\left\{w(a',t)-\int_0^\infty c(a,a',t)da\right\}.
\end{aligned}
$$

where D and e are positive constants. δ_{11} and δ_{22} are the constant natural death rates for male and female respectively. $M(a)$ is the *age-dependent* sex reversal rate from male to female at age a, where $M(a)=0$ for $a<a_c; m$ for $a_c\le a$. Just after the critical age a_c, the sex reversal from male to female could occur. Now we define $C(t)\equiv\int_0^\infty\int_0^\infty c(a,a',t)dada'$, $V(t)\equiv\int_0^\infty v(a,t)da$, $W(t)\equiv\int_0^\infty w(a',t)da'$ and $X(t)\equiv\int_0^{a_c} v(a,t)da$. We can then reconstruct from the above to the following dynamical system for the temporal variation of population sizes:

$$
\begin{aligned}
\frac{dV(t)}{dt} &= -\delta_{11}V(t)+\beta\lambda C(t)-m\{V(t)-X(t)\}\\
\frac{dX(t)}{dt} &= \beta\lambda C(t)-\delta_{11}X(t)-v(a_c,t)\\
\frac{dW(t)}{dt} &= -\delta_{22}W(t)+(1-\beta)\lambda C(t)+m\{V(t)-X(t)\}\\
\frac{dC(t)}{dt} &= -(\delta_{11}+\delta_{22}+D)C(t)+e\{V(t)-C(t)\}\{W(t)-C(t)\}.
\end{aligned}
$$

As for $v(a_c,t)$, we can derive the following result with the method of characteristic curve:

$$
v(a_c,t)=\begin{cases}\beta\lambda C(t-a_c)e^{-\delta_{11}a_c} & (t>a_c)\\ v_0(a_c-t)e^{-\delta_{11}t} & (t\le a_c).\end{cases}
$$

We assume that the initial population sizes $V(0)$, $X(0)$ and $W(0)$ are positive, and $C(0)=0$.

We analyze this population dynamics and discuss the influence of this type of sex-reversal on the dynamical nature, especially focusing the effects of the sex reversal rate m and the critical age a_c. In our analysis, we separately consider three cases in terms of the value of critical age a_c: ∞, 0 and finite positive. For each case, we analyze the stabilities of equilibria, and the asymptotic states as $t\to+\infty$, making use of analytical method and numerical calculation.

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