Stability Analysis of a Stage-Structured Time-Delay Model

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We proposed a time-delayed model of one species, which has a life history and has been exposed with Environmental Hormone. The individual members of the population will grow through two stages, immature and mature such as mammalian populations. The model is

$$\begin{cases} \dot{N}_{ai}(t) &= \alpha_1 N_{am}(t) + \alpha_2 N_{nm}(t) - \gamma N_{ai}(t) \\ & \quad -\alpha_1 e^{-\gamma\tau} N_{am}(t-\tau) - \alpha_2 e^{-\gamma\tau} N_{nm}(t-\tau) \\ \dot{N}_{am}(t) &= \alpha_1 e^{-\gamma\tau} N_{am}(t-\tau) + (\alpha_2 + \mu\alpha_3) e^{-\gamma\tau} N_{nm}(t-\tau) \\ & \quad -\beta N_{am}(t)(N_{am}(t) + N_{nm}(t)) \\ \dot{N}_{ni}(t) &= \alpha_3 N_{nm}(t) - \gamma N_{ni}(t) - \alpha_3 e^{-\gamma\tau} N_{nm}(t-\tau) \\ \dot{N}_{nm}(t) &= \alpha_3(1-\mu) e^{-\gamma\tau} N_{nm}(t-\tau) - \beta N_{nm}(t)(N_{am}(t) + N_{nm}(t)) \\ N_{am}(s) &> 0, N_{nm}(s) > 0, \text{ on } -\tau \le t \le 0 \\ N_{ai}(0) &> 0, N_{ni}(0) > 0 \end{cases} \quad (M)$$

We prove the positivity and boundedness of the solution and the uniform persistence of the system. The stability of equilibriums of the model is also studied. It is shown that under suitable hypotheses there exist stable nonnegative equilibrium, even there exist globally stable positive equilibra. The effect of the delay on the populations and the affection of environmental hormone are also considered. We find the population will survive although the environmental hormone gives some affection to their productivity. But if the environmental hormone become larger, the population will be doomed to extinction, that is, the positive points will be globally asymptotically stable under the difference condition.

References

[1] W. G. Aiello & H. I. Freedman, A Time-Delay Model of Single-Species Growth with Stage Structure,*Math. Biosci.* 101, 139-153. (1990)