A MODEL FOR BACTERIA AND BACTERIOPHAGE WITH TIME DELAYS

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Introduction

We study a model for resource-limited population growth and competition on three trophic levels: (1)primary resources,(2)first consumers or prey,and(3)predators.

We consider two cases in some detail. The first is the situation with one resource,one prey,and one predator. The second is the situation with one resourse,two prey,and one predator population.

The Basic Model

Let r_j be jth resource concentration, n_i are individuals per milliliter of the ith prey species (=bacteria),p_k are individuals per milliliter of the kth predator population(=bacterio phage) and m_{ik} are individuals per milliliter of prey of the ith type which have been attacked by a predator of the kth type and will give the birth of new predators.

Under these assumptions,the variables r_j, n_i, m_{ik} and p_k are related by the following delay differential equations:

$$\begin{aligned}
\dot{r_j} &= \rho(C_j - r_j) - \sum_{i=1}^{I} \phi_{ij}(n_i + \sum_{k=1}^{K} m_{ik}) \\
\dot{n_i} &= n_i \sum_{j=1}^{J} (\phi_{ij}/e_{ij}) - \rho n_i - \sum_{k=1}^{K} \gamma_{ik} n_i p_k \\
\dot{m_{ik}} &= \gamma_{ik} n_i p_k - \rho m_{ik} - e^{-\rho l_{ik}} \gamma_{ik} n_i(t - l_{ik}) p_k(t - l_{ik}) \\
\dot{p_k} &= \sum_{i=1}^{I} b_{ik} e^{-\rho l_{ik}} \gamma_{ik} n_i(t - l_{ik}) p_k(t - l_{ik}) - \rho p_k - \sum_{i=1}^{I} \gamma_{ik} n_i p_k
\end{aligned}$$

We restrict our analysis to what we have called the "equable mode" of existence,meaning one in which primary resources enter a habitat of volume Vml at a constant rate Wml/h and that the fluid containing the unutilized resources,organisms,and wastes is removed at same rate. l_{ik} is the latest period between the attack by predator from the kth predator population on a prey from the ith prey population and the resulting production of new predators. $\phi_{ij}(r_1, \ldots, r_j, \ldots, r_J)$ is some function of the concentration of the various resources. γ_{ik} is attack rate for prey and b_{ik} is the production of new predators of attacking type. Let $\rho = W/V$ be the rate of flow through the habitat.

We discuss that existence of the equilibria and local stability of the equilibria.

Reference

Bruce R. Levin,Frank M. Stewart,and Lin Chao, Resource-limited Growth, Competition,and Prediction:A Model and Experimental Studies With Bacteria and Bacteriophage, The American Naturalist 111 3-24