Possibility of Parasitoid's Invasion at the Host-Parasitoid Systems

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In this article we suppose two hosts ($H_1,\ H_2$), their two specialist parasitoids ($P_{S1},\ P_{S2}$), and a generalist parastoid (P_G) that is able to attack both hosts. From these species' interaction, the community's srtuctures are formed. The aim of our analysis is making the local stability of these structures clear.

$$\begin{aligned} H_i(t+1) &= \lambda_i H_i(t) f\left(P_{Si}(t)\right) f\left(P_G(t)\right) \\ P_{Si}(t+1) &= H_i(t) f\left(P_G(t)\right)\left[1 - f\left(P_{Si}(t)\right)\right] \\ P_G(t+1) &= \sum_{i=1}^{2} H_i(t)\left[1 - f\left(P_G(t)\right)\right] \\ f(x) &= \left(1 + \frac{a.x}{k}\right)^{-k} \end{aligned}$$

We assume that hosts and parasitoids have discrete generations, and three parasitoids attack host independently. Then these phenomenons are expressed by above difference equations for $i = 1,\ 2$. Where $H_i(t),\ P_i(t)$, and P_G represent the dencities of each species in the current generation. Let $\lambda_i\ (> 1)$ be the fecundity of host H_i. $f(x)$ implys the probability of a host escaping parasitism, and the dot refers to any of the three parasitoid species. This inherit from a concept of the zero term in the negative binomial distribution used by *Nicholson-Bailey* model. We assume that the searching efficiencies of the two specialist parasitoid are a_S and that the searching efficiency of the generalist is a_G. And the parameter k indexes the amount of density dependence in the risk of parasitoid attack.

When the two hosts have identical fecundities, the local stability of the community's srtuctures provide the following.

- **Structure (a)**
  We assume that $H_1,\ H_2,\ P_{S1},\ and\ P_{S2}$ exist in the communities. In this case, as magnitude of any eigenvalue at the equilibrium point are less than 1, structure (a) is stable.
- **Structure (b)**
  We assume that $H_1,\ H_2,\ and\ P_G$ exist in the communities. In this case the equilibrium points form a segment (l_1). Since the eigenvector of the eigenvalue 1 is $(H_1,\ H_2,\ P_G) = (1,\ -1,\ 0)$ and magnitude of the rest of eigenvalues are both less than 1 at the any equilibrium points on l_1, then a solution starting at the neighborhood of the segment l_1 converges to the segment l_1.
- **Structure (c1)**
  We assume that P_{S1} invades the stable community of structure (b). In this case the equilibrium points also form a segment (l_2) connecting $A\ (\hat{H}_1,\ 0,\ 0,\ P_G^*)$, $B\ (0,\ \hat{H}_2,\ 0,\ P_G^*)$. Let us consider a equilibrium point dividing the segment AB at a ratio of $(1-m) : m,\ (0 < m < 1)$. There are eigenvalues 1 and $\frac{k(\lambda^{1/k}-1)m}{\lambda-1}\Phi$. The magnitude of the rest of them are less than 1. Since the element P_{S1} of the eigenvector belonging to 1 is zero, invasion of P_{S1} to structure (b) is impossible. While as the element of P_{S1} belongs to the eigenvector equals to $\frac{k(\lambda^{1/k}-1)m}{\lambda-1}\Phi$ is non-zero. Hence invasion of P_{S1} to structure (b) is possible only if the magnitude of this eigenvalue is greater than 1. In other words, in the neighborhood of segment AQ structure (c1) is unstable, while in the neighborhood of segment QB structure (c1) is stable. Here Q is a point dividing AB at a ratio of $(1-m_0) : m_0$, ($m_0 = \frac{\lambda-1}{k(\lambda^{1/k}-1)\Phi}$). Moreover, the following is provided by the numerical analysis. We can observe that any solutios starting at the neighborhood of AQ converges to QB.

[Reference]

H. B. Wilson, M. P. Hassell, and H. C. J. Godfray. "HOST-PARASITOID FOOD WEBS: DYNAMICS, PERSISTENCE, AND INVASION". *The American Naturalist.* Vol. 148.